PERMUTATIONS & COMBINATIONS

Part I

COUNTING TECHNIQUES

PERMUTATIONS

- Permutation is the number of ways to arrange things.

  Eg : My safe code is 492.

  (order matters)

- $P(n,r) = {}^nP_r = \frac{n!}{r!}$, Where $0 \leq r \leq n$

  n → the number of things to choose from

  r → the number of things we choose

  ! → factorial.

COMBINATIONS

- Combination is the number of ways to choose things.

  Eg : My Salad is a Combination of carrot, Onion, Tomato and Lemon.

  (order doesn't matter)

- $C(n,r) = {}^nC_r = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$; Where $0 \leq r \leq n$

  n → the number of things to choose from

  r → the number of things we choose

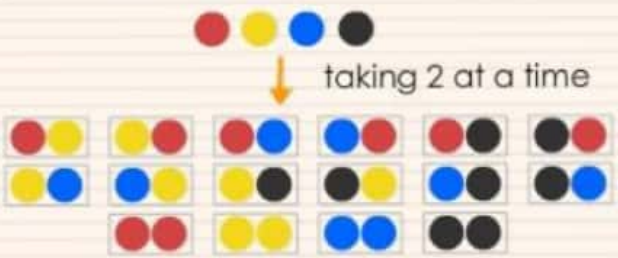
TYPES OF PERMUTATIONS & COMBINATIONS

When Repetition is Allowed.

1. Permutations with Repetition

Formula: n^r

(Repetition allowed, order matters)



2. Combinations with Repetition

Formula: ${}^{(n+r-1)}C_r$

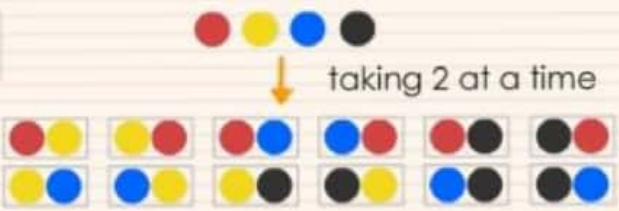
(Repetition allowed, order does not matter)

When Repetition is not Allowed.

1. Permutations without Repetition

Formula: ${}^nP_r = \frac{n!}{(n-r)!}$

(No repetition, order matters)



2. Combinations with Repetition

Formula: ${}^nC_r = \frac{n!}{r!(n-r)!}$

(No repetition, order does not matter)

Part II

PERMUTATION

If 'n' is the number of distinct things and 'r' things are chosen at a time.

1. Permutations of objects when all objects are not distinct.

$$\text{Permutations} = \frac{n!}{P_1!\, P_2! \ldots\ldots P_r!}$$

$P_r \rightarrow$ Number of things among 'n' are excatiy alike of r^{th} type.

2. Permutations with Repetition

Number of Permutations = n^r

3. Circular Permutations

Case 1 : When clockwise and anticlock wise arrangements are different.

Number of Permutations : $(n-1)!$

Case 2 : When clockwise and anticlock wise arrangements are not different.

Number of Permutations : $\frac{1}{2}(n-1)!$

4. Permutation under Restrictions

Case 1 : When 's' particular things are always to be included.

Number of Permutations : $\frac{(n-s)!\, r!}{(n-r)!\, (r-s)!}$

Case 2 : When a particular thing is always to be included (s = 1).

Number of Permutations : $\frac{(n-1)!\, r!}{(n-r)!\, (r-1)!}$

Case 3 : When 's' particular things are never be included.

Number of Permutations : $\frac{(n-s)!}{(n-s-r)!}$

Case 4 : When a particular thing is never included (s = 1).

Number of Permutations : $\frac{(n-1)!}{(n-r-1)!}$

Case 5 : When 'm' particular things always come together.

Number of Permutations : $(n-m+1)! \times m!$

Case 6 : When 'm' particular things never come together.

Number of Permutations : $n! - (n-m+1)! \times m!$

Part III

COMBINATION

If 'n' is the number of distinct things and 'r' things are chosen at a time.

1. Combinations with Repetition

Number of Combinations : $^{(n+r-1)}C_r$

2. Total Number of Combinations

Case 1 : Ways of selecting one or more than one things.

Number of Combinations : $^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n - 1$

Case 2 : When 's_1' alike objects of one kind, 's_2' alike objects of 2nd kind and so on ..... 's_n' alike objects of n^{th} kind.

Number of Combinations : $(s_1 + 1)(s_2 + 1) \ldots\ldots (s_n + 1) - 1$

Case 3 : When 's_1' alike objects of one kind, 's_2' alike objects of 2nd kind and so on .... 's_n' alike objects of n^{th} kind and rest 'p' different objects.

Number of Combinations : $[(s_1 + 1)(s_2 + 1) \ldots\ldots (s_n + 1)]2^p - 1$

3. Combinations Under Restrictions

Case 1 : When 's' particular things are always to be included.

Number of Combinations : $^{(n-s)}C_{(r-s)}$

Case 2 : When a particular thing is always to be included.

Number of Combinations : $^{(n-1)}C_{(r-1)}$

Case 3 : When 's' particular things are never included (s = 1).

Number of Combinations : $^{(n-s)}C_r$

Case 4 : When 'm' particular things never come together.

Number of Combinations : $^nC_r - {}^{(n-m)}C_{(r-m)}$